Nine Week Language Course for 6th Graders

Round 1: Some Languages (Three Weeks)

Predicate Logic: A Language of English and Math tying logic and quantification
Geometry 2D: A Language of Straight Edge and Compass
Algebra: A Language of relationship between number and letter
Algebra and Geometry 2D: A Language in Group theory tying Algebra and Geometry
Conics: A Language to Slice a Cone into Spin, Throw, and Hang.

Round 2: Examples of Language (Three Weeks)

Example 1: Truth in AdvertisingExample 2: Your Choice of Scale Drawing by 2Example 3: Computer programming for the best box.Example 4: Sticks, Spools, and a Vector TreeExample 5: Styrofoam and a Wizards Hat

Round 3: Group and Individual Cumulative Projects (Two Weeks)

Project 1: Smoothie on Over to Clay and Computer Project 2: Indoor/Outdoor Construction of the number 17

Final Round: Review and Test (One Week)

$$\rightarrow$$
 = - \leftrightarrow v \wedge

Introduction

Good morning.Did you know that that 'All Dogs Pee on a Fire Hydrant"?Oh, you think that's not true?Fine. I believe you.Let three volunteers to come up to the board and simultaneously, write the opposite.

This is the start to a three week course in Language, from, a mathematical perspective.

But this is an education course; and it is one that everyone will learn.

We are going to cover geometry, algebra, conic sections, and a little programming.

But we are going to start with how to communicate proper mathematical language.

You will be, the talk of home school.

This nine week course will emphasize clarity in communication between class members; that includes the Teacher.

We will set the rules for engagement of an idea.

We will come to a consensus as to the clarity of a written expression about that idea.

We will utilize the agreed upon rules of predicate logic to validate a stated idea.

And there is no one in this room who is not capable of clarity.

Some will be verbose, while others will be concise, but validation is indifferent to either.

We are individuals that have distributed patterns of thought, and that's ok; no, its really ok.

We will learn this week, how to voice an argument, write down the argument, then induce the rules of logic to see how the argument fairs out.

The most important thing to take away from this lesson, is the concept of antipodal pts. That is, under what conditions is the argument sound and unsound.

And then we can move on to easier things like drawing, multiplying numbers, and programming computers; the latter being the easiest.

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Predicate Logic

Part one: Logic

A Language of English and Math and minding your Pea's and Que's

Some Basic Rules of Engagement

Rule # 1.

A 'statemented idea' is something that we can agree upon as either TRUE or FALSE. Sometimes, in order to proceed, we will note disagreements.

Statement 1: "The sixth grade science book has a lot of pages". How do you measure ' a lot of'? Well, we will soon enough. Hey, where's the period go, before or after the "?

Statement 2: How about "The sixth grade science book has 139 pages"? Can't get much clearer. Really no argument here. Hey, am I missing a .?

An idea, may require one or more 'statemented ideas'.

S3: So what about "The sixth grade science book has 139 pages and that's a really Really huge big-time lot of pages." ? Know what? Let this be just 'Predicate Logic', and not 'Adjective Predicate Logic". So let's rewrite S3: "The sixth grade science book has 139 pages and that's a lot." ? To repeat:

S3: "The sixth grade science book has 139 pages and that's a lot."?

Do you think there is more than one statement here?

The first problem: How many parts are there to this complex looking statement?

We will need to agree on how to break apart a complex statement.

Then, we will need to agree that each statement can be labelled as True or False.

This is , in my opinion, the hard work.

We will have as exercises, statements that at some point we must all agree on their being True or False so that we can then test out the Logical validation.

Note # 1.

Let's agree to shorten every statement into something on the order of a single letter as a symbol to take the place of the statement.

But, let's try not to use the letters T and F in place of statements simple because, as was alluded to in Rule #1, in predicate logic we are going to evaluate and conclude as to whether the statements or combinations of the statements or True or False, i.e., T or F.

Letters like P, Q, R, S are generally used. For statement 1 and statement 2 we could simply have

P=: "The sixth grade science book has a lot of pages".

Q= "The sixth grade science book has 139 pages".

With a new statement, say, "The sixth grade science book is yellow", we could let: Y= "The sixth grade science book is yellow.

Or we could just as easily use S1, S2, and S3 instead of P, Q, and Y; i.e., indexing a single letter.

What are naming these statements: 6=1+1+1+1+1+1 6=1+1+1+1+1+1+1 How about S5 for the first one and S6 for the second one. M and N would be fine too.

Our Basic Field Play Rules of Predicate Logic

Rule # 1 – In Predicate Logic, a predicated statement must be agreed upon to be True or False.

Note # 1 – Predicate statements will be represented by single letters like Peas and Ques.

So in review when we state "The book in my hand has 539 pages", we may shorter this to X. This X will be either T or F.

A question for the English teacher, "Are we making a more 'concise' set of statements when we are incorporating Mathematical logic symbols along with the language of predicates".

(Hey, did we ask if the quotation marks go after or before the period.)

One more example: We will have in-class and take-home exams with these rules. I see IC is T and TH is T, or C is T and H is T, or P is T and Q is T.

The Basic Truth Tables

Ρ

Т

Т

F

F

0

Т

F

Т

F

A table has the purpose of listing all combinations Ρ Of True and False. Let's start with a single statement, call it P. Т A table for P has the statement symbol on top and, F the True \rightarrow T and False \rightarrow F below like this: It's a slow start to something that grows quick. Now, what if we have two statements, say statement P and statement O? In this case, we have to account for four different scenarios, i.e., we need to account for when the Pea's and Que's are true or false at different times. (Hey, could we write 'true and false'?) And, it is when we have two or more statements, that we really start to form a cool looking truth table like this one: Now, for the average guy, like me, I just have to do a double take, just make sure the rows are unique. BTW, girls seem to be better at setting up the tables right, the first.

When we start combining statements to form an idea the tables become indispensable.

With any two statements, there are the three basic logical combined statements.

Conjunction, Disjunction, Conditional.

Can we all say together....Huh? Well, ask your English Teacher for help here. But basically, these are the 'not-so-basic' rules we will use a lot.

Now, just as we have symbols for statements we will have a symbol for each of *Conjunction, Disjunction,* and *Conditional.*

And, these symbols are recognized World-wide; it's the basis for universal mathematical Language.

That's the easy part.

Agreeing as to whether a statement P is T or F is sometimes hard enough without Further complicating things with the 'shuns' above.

Now you see...

Conjunction

A conjunction is a conclusion about the truth of two combined statements. Think of 5 being the conclusion to 3+2.

Think of that + sign as being the CONJUNCTION to the two statements 3 and 2. Huh?

Yes, people, it's deep.

Now, another term for **CONJUNCTION** is the word **AND**. When writing a statement you certainly may use either at any time.

For **CONJUNCTION** (or **AND**), we have the symbol : Λ .

So just like 3+2 we have $P \land Q$.

Here is the table with conjunction added:

| Ρ | Λ | Q | |
|---|---|---|----------------------|
| | | | And now what? |
| Т | | Т | We fill in our first |
| F | | F | real table 🔿 |
| Т | | F | |
| F | | т | |

Rule # 2 is for Conjunction

Here it is, y'all been waiting for, with great anticipation, Your first filled table when there are two statements. four red letters right under the conjunction.

And now, we will come to all agree as to the meaning Of these letters,

and this ain't meant to be rocket science nor quilting.

(Hey, Is 'ain't' a real word ?.)

(Hey, May I or May I not have a period after question mark?) ((Excuse me, can we focus here a little more, please?))

So,

our **conclusions**,

in red,

are again,

either True or False.

And this is accepted world wide.

Let's read through the rows, one at a time, and think.



Looking under the AND symbol, these red T/F's are the agreed upon outcomes when the conjunction AND joins P and Q. The entries of P, Q and AND are aligned horizontally. So look at the third row Down and

you read a T under P, an F under Q, and a red F under Λ . The red F is the agreed upon result of the conjunction.

That is, Given the statement P AND Q (It is raining AND I have an umbrella in my locker to give you), and later we find out that that it is raining but there is no umbrella, then the statement is false.

The tricky case is when both P and Q is false.

You might tend to believe that two negatives make it true (like -3 * -4 = +12), but that's not what the agreed upon outcome to this conjunction is.

Now, there is room to argue ahead of time as to whether or not the pea's and que's are T or False, but there is no arguing what the AND symbol represents, the RED outcomes are the so-called law s of Logic.

| Ρ | Λ | Q |
|---|---|---|
| | | |
| Т | Т | Т |
| F | F | F |
| Т | F | F |
| F | F | Т |

You see,

later on, (Let's put a little math happy expression into these sentences people) actually,

very shortly,

we will be creating a set of compounded statements,

But we will not be able to know ahead of time if the basic statements, as in statement P by itself, or statements Q,R,S,T,... by themselves, are in fact true or false.

At best we will agree that the statements are written down such that we can agree that when we get around to the opportunity that the statements can be proved true or false noting that

this is where the vagueness in contract/legal writing comes into play, and yes, even in Math. But math is,

a discipline of writing out things clearly,

in very dry appearing symbol formation.

However, we need a language to write down the language expression, and then,

we need science, or History,

to help us verify the facts about the statements,

and then we need Philosophy,

to dive a little deeper,

and you can see where things get passionate and volatile.

And it makes things interesting, as you will find out, when its your turn to promote a series of statements, before your classmates, that you will prove, outright, based on the mathematical rules, we are finally ready to go use, but only after, we agree on whether the statements are provable to be true or false, not whether they are in fact true or false.

Ok, next 'shun' The next combiner of statements is *Disjunction*.

Disjunction can equivalently be replaced by a more common term, 'OR'. OR contrasts to the AND but it is not the exact opposite of AND.

I mean, 'conjunction' and 'disjunction' sound opposite, right?

Let's get philosophical for the remainder of the period and fill in the table below. It's generally called a Truth table by the way, even though the answers under the OR can be either True or False.

The point is, the evidence of intended truth or of non-truth is in this table.

| Ρ | V | Q | | | |
|---|---|---|--|--|--|
| | | | | | |
| Т | Т | Т | | | |
| F | F | F | | | |
| Т | Т | F | | | |
| F | Т | Т | | | |

So, a Disjunction/Or is only false when BOTH statements are False. When comparing the Conjunction/And to the this one, make sure the Entries under the P and Q are aligned by the row. Next..., a little tricky here, but you can do it.

You know how when we pose the math expression 1+4+6 we can add the numbers in a couple ways say add the 1 and 4 to get 5 and then add 6 to get 11, or Some people like to look for easy tricks like adding 6 and 4 first because they know it is 10, and then add 1 to get 11. For me, with 7+5+8, I am going with 7+8 to get 15 and Then add the five. But then again what are you going to do with 6+7+2+5? Maybe just Go straight across taking the first two, then the last two, and then add those two sets. (ooh 13 + 7 = 20). Anyway, it's about the same for the case when there are more than two statements. So let's figure it out and agree on how to fill out the T/F's, both for the T/F's that go under the statements, and then, for what goes under the conjunction, disjunction, and conditional symbols (no you didn't miss the conditional, we haven't covered it yet, I know you are anxious! Just keep still!)

Step 1, insert T/F below the A,B,C statements, We needed, two T/F's for One statement, and four T/F's How many for three? Let's decide that now. Step 2, treat A V B V C like AV(BVC) which means do BVC first, Then take that answer and OR it With A



Now for this one:

| Α | ٨ | В | ٨ | C |
|----------|---|---|---|---|
| _ | | | | |
| | | | | |
| | | | | |
| | | | | |

And now for this one:

| А | ٨ | В | V | С |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

And for the heck of it, let's have one-half the class start with A AND B, Then bring in C, And other half start with B OR C and then bring in A. So with 3+4+5 it does not matter which pair we start with, it's because we are adding. Now with 5+4*3,

it does depend on what pair you choose first,

remember?

There is the convention that multiplication takes 'precedence' over addition so, technically,

there is only one way,

do the 4*3 first to get 12 and then add 12 to 5 to get 17.

And like wise there are conventions for precedence with the respect to our three 'shuns', *Conjunction, Disjunction, Conditional.*

But in this class, we will not play that game. We are discussing real problems, written down in statements, which we have to agree upon, In which we impose rules of T/F logic, to drive toward combined statement outcomes, let's not Leave this to some convention when we can mitigate confusion by simply, very simply, Installing parenthesis, to direct our meaningful intention. I thank you, I thank you very So when we see the following written,

A V B Λ C we will ask to install parenthesis to be clear, and that way, we know the Person who originated of the statement has it interpreted the way it was intended, thus

we will install parenthesis according to (A V B) Λ C or A V (B Λ C).

So, let's ignore the questionable complex statement without parenthesis and put down the answers for



But for the fun of it, how about these two questions ?

| 1. | [(A | ٨ | B) | V | C] | V | [A | ٨ | (B | V | C)] |
|----|------|---|--------|---|-----|---|---------|---|--------|---|----------|
| 2. | [(A | ٨ | B) | V | C] | ٨ | [A | ٨ | (B | V | C)] |

These problems beg the question, exactly how many T/Fs Do we need under the A,B,C's (?).

3. How would an English teacher write out either of the above statements?

4. Now try this one:

"Either it was raining and I had an umbrella to give you or I like sitting in the rain and I was counting the raindrops."

There are four statements, two ANDs and an OR.

And finally, here is a third combiner, the CONDITIONAL statement: Say P = The car is driving fast, and Q = the Bridge is closing. We will consider the two statements:

- 1. IF the car is driving fast THEN the bridge is closing.
- 2. IF the bridge is closing THEN the car is driving fast.

They are not the same.

I think we need to talk to the English teacher to discuss a rewording of the statements,

Say for statement # 1, is it equivalent to write

"THEN the bridge is closing IF the car is driving fast"

(generally, by the way, the THEN would be dropped as a matter of speech protocol).

In terms of math symbols, we have:

- 1. P -> Q
- 2. Q->P
- 3. Where the arrow takes place of the IF/THEN

(Note the change in order of T/Fs under the peas and cues)





Here it is again,...



Ok,

so get this,

if you start out with a false statement,

it doesn't matter whether the ending statement is false or true,

the answer in red,

says the combination of the statements,

When using a conditional,

is true.

And that's the rule.

And it may be open to philosophical debate, law proceedings, but that's the agreed upon conclusion, world wide.

Now consider combining statements 1 and 2 with a conjunction

IF the car is driving fast THEN the bridge is closing AND IF the bridge is closing THEN the car is driving fast.

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In symbols we have : P \rightarrow Q \land Q \rightarrow P, or more clearly, (P \rightarrow Q) \land (Q \rightarrow P).
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Let's do the truth table, get the results,
and again,
get a little philosophical,
And come to terms with its deeep
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(Hey, is it better to say 'and more clearly' rather than 'or more clearly'.) (Hey, is there a legitimate way to include those extra eee's?) Note: one must remain organized; the Peas and Cues have the corresponding T/Fs below them on EACH side of the AND



Question for english teacher, in the context that the AND conclusion is false under two scenarios; namely, the T/F conditions under the P's and Q's for the2nd and 3rd Rows; which do we say that the overall combined statement is false when: (1) When P is True and Q is False "AND" When Q is true and P is false or (2) When P is True and Q is False "OR" When Q is true and P is false or is all of this drilling down too much and useless nit-picky-ness?

It may seem a little tricky, it is. But be clear, take your time, double check, And do it right. Your business will depend on it and besides, This is not rocket science or quilting.

Quick question for the Englsh teacher,

Is the following a truth-tabler?

Quilting is easier than truth tables? (we will get to quantification in the next topic, again, hold on to your enthusiasm, and please remain in your seats.

By the way, when you see

$(\mathsf{P} \to \mathsf{Q}) \quad \bigwedge \quad (\mathsf{Q} \to \mathsf{P})$

We can shorten this to

 $P \leftrightarrow Q$ Or equivalently $Q \leftrightarrow P$ Here we say: if P then q and if q then P Or very short: <u>P if and only if Q</u> (noting P if Q can be written as if Q then P, leaving P only if Q meaning if P then Q

By the way, I knew you were wondering. How would the table be done?

Now, let's have some fun and do excercises, Then we will talk about quantification and learn how to Take the opposite of "All Dogs Pee on a Fire Hydrant", And then we will do Laurie Sheen examples. Example:

If either you buy some milk at the store or fill up the gas tank in the car, then you will get some supper.

Example: Our unfinished business: S3

Predicate Logic

Part two: Quantification

A Language of English and Math and eating some or all of your Pea's and Que's

Baby Quantification

Categorical Propositions: Universal and Particular

Example:

1) All politicians are liars

2) All politicians are not liars (Note: same as No politicians are liars; check w/english)

- 3) Some politicians are liars
- 4) Some politicians are not liars.

Why the space between 2 and 3? Because,

1 and 2 are known as 'Universal' affirmative and negative (respectively)

3 and 4 are known as 'Particular' affirmative and negative (respectively)

Does this mean that 1 and 2 are opposites; and same with 3,4?

No.

Given the quantification statement # 1, either # 3 or #4, is the opposite, i.e., negation.

Answer: the understanding is that 1 and 4 are negations of each other, and 2 and 3 are negations of each other. Let's express this in symbols ...

The symbols look a little strange at first but later on they make complete sense.

We have something to say about ALL X, so what letter could we use? Yah, A. Except, A stands for apple so, all we do, to make look mathematical, is turn it upside down Like this: \forall .

Now what about the word 'some', which, by the way, I usually substitute with 'there-exists'. And even more technical and cleaner, 'there-exists-at-least-one'.

So while 'some' may imply more than one, it can mean exactly one or at least one, but Definitely, More than none! Now we could use S for some, but we don't. We actually, in the math world, like the 'there-exists, so we use E for exists, but whoa!, it's math, it's gotta be cooler den-dat. Now flipping it upside down like the A, would still give you a normal E, so in

This case, we flip it left/right to get a backwards E, like this: \exists

So, if we interpret p(x) to be a statement about a thing called x, we have the following math symbols with their negations in place.

Understand that $[\forall(x): p(x)]$ means, "for all x-things, here is the statement about these x-things. Using #2, All politicians are liars, we have ALL, the x-things are the politicians, And the statement we want to make is that they are liars. Now we write out the negation:

 $\neg \left[\forall (x) : p(x) \right] \equiv \exists (x) : \neg p(x)$

The minus sign with the hook is the negation sign, meant to be slightly different from the Normal minus sign.

Anyway, the opposite of, or the negation of all politicians are liars is: There exists at least one politician who is NOT a liar., ie., some politicians are not liars.

Now technically, could it be the case that 'ALL of them are not liars", and we would have to change the backwards E to the upside down A, but technically, that statement Is too restrictive, for a reason, we are only say there is undeniably at least one who is not and we will go about the more restrictive case later on.

(Hey, doesn't restrictive almost seem like a cool legitimate word?) (LOL)

So ALL goes to THERE EXISTS (SOME) when negating.

Now what if we start out with :

 $\exists (x): P(x),$

It's negation is understood to be,

 $\neg [\exists (x): P(x)] \equiv [\forall (x): \neg p(x)],$

So given "it is not true that some politicians are liars', we have the equivalent way of saying it with "For all politicians, none are liars". And there are other ways of saying it, ask English.

Ok test time: What is the negation of, wait for tit, ready? "All dogs pee on fire hydrants". Review and homework.

Q

F

0

Т

F

F

Т

We started with a single statement, called P. P is provable to be either true or false.

A conjunction is a statement about the truth of both statements;

Ρ

Т

F

Т

F

Ρ

Т

Т

F

F

Т

т

т

F

Disjunction is the Or statement.

the CONDITIONAL statement:

Biconditional:

 $(P \rightarrow Q) \quad \Lambda \quad (Q \rightarrow P)$

∧ Q _____ T T F F F F F T

Ρ

Т

F

Т

F

T F Biconditional: $(P \rightarrow Q) \land (Q \rightarrow P) \equiv P \leftrightarrow Q$

Quantification

 $[\forall (x): p(x)]$ means, "for all x-things, here is the statement $[\exists (x): P(x)]$ means, for some things x or there exists at least one x

$$\neg [\exists (x): P(x)] \equiv [\forall (x): \neg p(x)],$$

$$\neg [\forall (x): p(x)] \equiv \exists (x): \neg p(x)$$

$$\neg [\exists (x): \neg P(x)] \equiv [\forall (x): p(x)],$$

$$\neg [\forall (x): \neg p(x)] \equiv \exists (x): p(x)$$

Homework: All dogs pee on fire hydrants
Now check to see equivalence -(PVQ) == -P and -Q etc

If either you buy some milk at the store or fill up the gas tank in the car, then you will not eat supper

Joe Jill ane and Jim are all drivers going for a ride in the same car. Joe gets nervous when jill or jim drives. Jim gets nervous when he or Jane drives. Jane gets nervous when joe or jim drives, and jill gets nervous when anyone other than herself.

1. Which driver makes the greatest number of people nervous? A Joe, B Jill, C Jane D Jim E Joe and Jill equally, F Jim and Jill equally

2. Which driver would make the least number of people nervous?, Joe, Jill, jane, Jim, Jill and Jane equally, Joe and Jim equally

3. Who should drive if jim is not to be nervous? Joe, Jill , Jane, Jim, Either Joe or Jill, either Jane or jim

4. Who should drive if neither Jane nor Joe is to be nervous? Joe, Jill, Jane, Jim, either jane or Joe, either Jim or Jill

5. Jill has too much homework to do, and she decides to not go for a ride. Now which Driver would make the least number of drivers nervous? Joe, Jil, Jane, Jim, Joe and Jane equally, jim and jane equally

6. Joy decides to go in place of Jill. Joy gets nervous only when joe drives and jane Gets nervous when joy drives. Now, which driver would make the least number Of people nervous? Joe, joy, jane, jim, joe and joy==, joy and jane ==

Geometry 2D

A Language of Straight Edge and Compass

The Basic Constructions of Geometrical Language using Straightedge and Compass

Two rules:

1. We agree that given two points, we can draw a straight line segment through them, with the straight edge.

2. We agree that for a single given point O and a given length r, there is exactly one circle that can be drawn through O with radius r, or just sub-arcs of the circle.

Construction 1 Given a segment AB, Construct a segment congruent to AB

Procedure:

1. Use a straight edge to draw a line longer than AB and place a point, call it X on the left end. With compass, set distance from A to B. Then set the compass point on X and draw an arc through the new line. Call the intersection point Y. Thus XY==AB.





construct another angle congruent to it.

Procedure

 $R \longrightarrow Y$

Draw a ray. Label it RY.

Using B as center, and any radius, draw an arc to intersect both sides of the angle. Label the intersecting points



Now using R as center, draw an arc through Y and extend it for more than angle ABC takes up. Label the intersect point S



Now with point of compass on S, Draw same arc through the arc That goes through S. Label the point of intersection T.



Now draw ray through RT, And we have the two congruent angles.



Construction 3 Given an angle, bisect it.

Given angle ABC, with B at the vertex (center)

Α

draw an Arc covering BA and BC. Call the pts of intersection X, and Y



Now here's the trick, from X and Y we Draw two more arcs, one arc uses A as the center and the other arc uses B as the center. Now these two arc, drawn like you see here, will intersect. Call that point Z. Then use the straightedge to draw in line segment BZ, this is the bisector.

So angle ABZ ==angle CBZ



Construction 4

Given a segment AB, construct the perpendicular bisector of the segment.

Easy way: Spread compass out about 2/3 the way from A to B. Then from A draw a circle. Do the same from B, with the same compass spread. Connect these points of intersection of the two circles, top and bottom. Connect the points, and you have the perpendicular bisector. Now, once you get good, you only needed to draw partial arcs above where you They cross the center, each from A and B. Try it for home school work.



So, we have constructed two things: The 90 degree Perpendicular Angles: AEC,CEB,... And the line segment bisector: AB is equally cut into AE and BE.

Anything else?

Construction 5

Given a point C on line k, construct the perpendicular to the line k, that goes Through C.

Ready? Here's the line with pt C. ooooh, I drew it at an angle. Hey, how do you Spell oooh? I want Who, with out the W. I don't want O. Anyway...



Gee, I didn't put in all the steps in this construction. People, my time is valuable, I have facebook and insta-grams coming in all the time. I can't keep up!

Construction 6. And this is cool. Given a line k, and a point off that line, call it D (or you can name it anything you want, I picked D because its my lovely wife's name, Denise.

Now again, I twisted the line a little.

Here we go.



WithcompasspointatDdrawarcsthecrosskdownbelowwithoneontheleftandoneontheright. Newsentence:ThenfromthesetwopointsdrawarcsbelowlinekwellrespecttoDwhichisabove LinekandwherethesetwoarcsintersectbelowmarkthepointofintersectionlabellingitEandthen DrawinlinesegmentDEandyouhavedrawntheperpendicularlinethroughkfromDandyouhave Improvedyourreadingskillsmagnificiently. Seeanyspellingmistakes? Construction 7. Is, in my opinion, tricky, and really cool to show at pool parties. Hello, bring sidewalk chalk and string, how hard is this? Well, let's see.

Construction 7 Given a line k, and a given point H, we are going to construct a line through H and Parallel to k.

Now before we begin, let's appreciate what's going to happen. Below I have drawn two sets of lines and points. On the left draw down the Perpendicular line from H to the line. Go ahead, be brave. Now on the right side Draw a line that is parallel to k but through H. Now which one do you have more Math faith in? Try this power point. Not that easy. Construction 7 continued.

Ready step 1. label two points, A and B on k, make one a little to the left under H And one to the right, a little farther out, For the effect mostly. Then draw a line thru HA, and extend way above H. Did I tell you why I picked the letter H?

Can we focus on Math people?!

Ok, now what?

Well, it's construction 2 to the rescue.

(1) From A, draw an arc that intersect AH
Somewhere in the middle and goes down
Through AB. Then, (2) with the point on H,
Draw the same kind of arc, and imagine,
Yes use your imagination people, that there
Is a line that's out from H parallel to B, and go
Through it. (3) From the point intersecting AH from
The first arc, draw an arc that will intersect the
Arc#1 that goes through AB. You have to set the
Compass To match the distance from the point on
AH to the pt on AB. Now do the same on H. And now
you have two points to draw the other side of the angle
Like in construction 2, but BONUS, the new line is parallel



Like in construction 2, but, BONUS, the new line is parallel to AB. That's right people, applause time!!

Theorem 1.

Build up to theorem 1.

All theorem one is, is an observation, After using a mixture of two constructions. The constructions are the basis. The theorem Will be a proof of an observation someone Made while experimenting, or day-dreaming In class, or at home, or waiting for your hair appt.

So given triangle XYZ, first construct the Angle bisector of each angle. The bisectors Will cross paths inside the triangle.

Then, from the point(s) they cross paths, Draw a perpendicular line to the sides XY, YZ, XZ. Y

Theorem 2.

The bisectors of the angles of triangle QRS Intersect in a point that is equidistant from The three sides of the triangle.

It works on any triangle.

Try this one.

Hey, first write an if-then statement, please. See, math is a polite language. So on the other hand,

Theorem 3.

The perpendicular bisectors of the sides of a triangle

Intersect in a point that is equidistant from the three vertices of the triangle. Write the if-then statement, pretty please.

Oh, math is very polite.

Is this the same as theorem 1? Let's agree on Why or Why Not.

Fall out Theorem 3.

The lines that contain the altitudes of a triangle intersect in a point.

I don't get this one: Theorem 4, The medians of a triangle (midpoints of each side, I think), intersect in a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side. Wha! Glad that came out! Back to Constructions. I know you missed them.

Construction 8 Given a point on a circle, construct the tangent line to the circle at the given point,

Procedure: Draw OA extending past A a distance more than OA. Now construct a perpendicular To this new line segment using construction 4. (Arcs on the Line segment to the left and right of A, and then Arcs from the intersection points, above and beloe A. Yup, very clever, the perpendicular line is also tangent To the circle 0 at A.



Construction 9. Ok getting a little challenging now, More than the parallel line.

Given a point outside a circle, construct a tangent to the circle from the given point

Procedure:

- 1. Draw OP
- 2. Now find the midpoint of OP call it MM.

MM is for Merry Midpoint.

Oh, wait, MM is one point and that's

Confusing with a segment like AB, but then again

I guess segment AA would be a point after all.

3. At M (let's stick to one letter), construct the perpendicular

 \cap

Ρ

bisector of OP through M. (using which construction?)

3. Now, using M as center, with radius MP on compass, Draw a circle centered at M

It intersects circle O at two points, call them x and Y.

Connect each of these points to O,

I rotated my paper. The constructions go: Red Green Black Purple Yellow

And what we are stating is that PX is tangent to OX, And same with PY and OY, thus OXP == POY=90. You believe that? Ya know, they Teach a course called Geometry in high school, I think we need a field trip to high school.



Construction 10.

Given a triangle, circumscribe a circle about the triangle.

This is easier done than believed too. See, this geometry is a really powerful В Language, when it comes right down to it. Procedure: Simple construct any two sides and Construct each of their Perpendicular **Bisectors**. They will intersect inside the Α Triangle. Then, from the intersection point, stretch your compass to any one vertex And start drawing a circle, and watch it go through All three vertices. Pretty wild, I think anyway, But then again, my instagram is backing up.

Construction 11 Given a triangle, inscribe a circle in the triangle.

Same triangle, just rotated to Keep you from being bored. In effect, we will place a circle inside The triangle, such that it will Touch each side of the triangle, once.

Now let's think about this. If you were asked to draw a circle inside this triangle, and For it to touch the sides exactly once, How close could you come?

Here is my try...



Real nice, right?

Let's do better.

Procedure

This time bisect any two angles. Then label their intersection, D. Then, from that point, Construct a perpendicular, to Any one of the sides. Then, adjust, the compass from D to the perpendicular intersection point.

Well, my red circle, doesn't really look that round, but, that's Why we construct, it works Doing it by hand, or computer.

В



Construction 12

Given a segment, AB, divide the segment into a given number of congruent parts.

Procedure: For this example, we will Cut AB into three segments.

1. Here's the trick.

first, pick a pick a point off of AB, Call it C, and draw segment, AC.

2. Then set your compass small enough to mark out, A to D, then D to E, than E to F.

(Red, Purple, Green).

3. Now connect F to B and then construct a line
Through E parallel to FB, and then the same through D.
Pretty good construction.
Better than facebook?
You know it is, deep down.
That's o, exhale now.



Arithmetic and Algebra

A Language of relationship between number and letter (using boxes and triangles) Ok, here we go.

```
Consider 4 + 3, really, if you use your fingers, like we had fun doing
in 1<sup>st</sup> grade, and 2<sup>nd</sup> grade, and ok, yah, may be 3<sup>rd</sup> and 4<sup>th</sup>.
Are there any 5<sup>th</sup> graders out there?
Anyway,
You could have
1 + 1 + 1 + 1 on one hand, and
1 + 1 + 1, on the other hand.
And you would get, in your mind,
(1+1+1+1) + (1+1+1) = 1+1+1+1+1 = 4+3 = 8
```

Up to now, we have always been focused on the '7', oooooh, because It's the answer, ooooh!.

Well, we are in sixth grade, so we are more mature than that, we want to 'experience the process' of getting the answer, not so much the answer. (but, if you do get the answer wrong, there is no 'my bad'. It's wrong.

What if we go around the room and everybody uses their fingers on both hands to come up with 7 total.

Ok, show me your fingers. And let's keep tab of the different types. First of all, does everyone have clean fingers? Here is one rule for now: Hold your fingers pointing up.

Now to get 7 fingers using both hands:

We could have Left hand have 2 and the Right hand have 5. That is: L + R = 2 + 5 = some answer. But what did we just do? I sneaked in L and R as algebra symbols for 2 and 5. Can the past tense of sneak be snuk?

Anyway. So why not just use 2 + 5 ? And that's a fair question. Well it's a matter of clarity and actually forming a one size fits all.

Now how is the 7 arrived at? Well, we are RELATING the two numbers 2, and 5, Or the algebra symbols, L and R through addition. The **relation** says to take two things and 'do this' to them., i.e., take 2, and 5, and addem', i.e., take L and R, and addem'.

Now in the first case, with 2 and 5 related, we know ,from say at least 2nd grade, we get 7 fingers. But in the second case, with L and R related, what can we do?

Right, how about L + R = T, where since we are not using truth tables, we can use T for TOTAL.

Now I want to use a symbol for RELATION. Since R is too common a symbol in algebra, we need another symbol. There is one that is used universally, and here it is: \mathcal{R} . Kind of purty, ain't it, huh? Let's practice :

R R R R R R ...hey I'm am getting good at this (Script, MT Bold)

So, to relate 3 and 4 I would write $\mathcal{R}(3,4)$, where \mathcal{R} takes the place of +.

Now, from around the room, we got 3+4, 2+5, but also 4+3, and 5+2. Ok, so we can write in algebra terms \mathscr{R} (L,R) when we have (L,R)'s = (3,4), (2,5), (4,3), and (5,2).

Algebra and Geometry

A Language in Group theory tying Algebra and Geometry Binary relations Reflexive : aRA reflexive Symmetric : If whenever aRb then bRa Transitive : if whenever aBb and bRc then aRc

```
Example let R = 'is parallel to'
```

Equivalence relation is when R is Reflexive, Symmetric, and Transitive.

Devlope understanding of relations with :

- 1. R = 'is less than'
- 2. R = 'Is the square of'
- 3. R = 'Has dinner with'

| SETS | | |
|------------|-------------|-------|
| No lesson, | just do it, | Nike! |

Let U ={a b c d e}, A={a b c} B={b d e} C={3 4 5 6} \cap Find: AUB, A ∩ B, (A ∩ C)^c, B\C, A^c

Π

Π

A group is a set G of elements with a binary operation (two things at a time)such that a*b is in G (a*b)*c=a*(b*c) There is an identify I such that a*I=a For every a there is an inverse such that a*a=I

A simple example is using an equilateral triangle , that is to be constructed, with composition as the binary operation





Let Rot(x) mean rotation ccw x degrees Let Ref(a) mean reflection in Line a

If we turn once all the way around, like the earth, we say 360 degrees. Why not 400?

By using 360 degrees, then we can have Rot 0, Rot 120, and Rot 360 Also, we can have Ref a, Ref b, Ref c.

Given the number 1,2, and 3, where do they end up after the composition. Here's the trick:

What are the elements? (the vertices after each type of rotation and reflection is applied). What is the binary operation? Composition, and in this case either rotation or reflection So what do we mean by a*b man? Finish this by showing all sets of vertices.





Example: Where do the jellyfish, penguins, and koala bear all end up being after: Rot 120*Ref a * Ref b

Note there is an interactive website for geometry: http://www.scienceu.com/geometry/facts/solids/java/html

Round 2: Examples of Language

Example 1: Truth in AdvertisingExample 2: Your Choice of Scale Drawing by 2Example 3: Computer programming for the best box.Example 4: Sticks, Spools, and a Vector TreeExample 5: Styrofoam and a Wizards Hat

Example 1: Truth in Advertising
A deeper, more formal look into **Predicate Logic**.

Predicate logic is the study of methods and principles used to distinguish correct reasoning from incorrect reasoning. Examination of fallacies

Vocabulary

Argument: any group of propositions of which one is claimed to follow from the others **Propositions/premises**: a true or false statement in which a fact or an opinion is stated **Conclusion**: a proposition which is affirmed on the basis of other propositions

Three types of arguments

Obviously genuine – the dispute where there is no ambiguity present among the disputers and there is an argument in belief

Merely verbal-the dispute where there is ambiguity present among the disputers and there is an agreement in belief

Apparently verbal-is the disagreement where there is genuine ambiguity present as well as the disputers disagree in attitude or belief.

Usage of language in three ways: informative, expressive, and directive.

Informative - used to affirm or deny propositions Expressive – used to show or express the arguer's feelings Directive – used to prevent the causing o over actions

13 Fallacies of relevance

Argument ad baculum – appeal to force, or where one appeals to the force or threat to cause acceptance of a conclusion.

Argument ad hominem – abusive, or where an argument is directed to a man and attacks the opposition position.

Argument adhominem-circumstantial, or where an argument attacks a person's belief and circumstance

Argument ad ignorantium, an argument where the premises are drawn from ignorance. **Argument ad misericordiam**, where an argument appeals to the pity of the listeners. **Argument ad populum-** or the appeal to the people to win assent by unsupported conclusion

Argument ad verecundiam – or the appeal to authority. This is an argument where the premises rely on the knowledge of a known source.

Accident-is the applying of a general rule to a particular case whose 'accidental' circumstances render the rule inapplicable.

Converse accident – or a hasty generalization is incorporated in the premises False cause

Petitio Pricipii – this is also known as begging the question. In this fallacy, the premise and the conclusion mean the same thing.

Complex question – in this straight forward answers do not answer question

Ignoratio elenchi – ths is also known as irrelevant conclusion. The argument is purporting to establish a particular conclusion and is directed to proving a different conclusion

5 fallacies of Ambiguity

They occur in arguments whose premises and conclusions contain ambiguous words or phrases

Equivocation – used when meaning of a word or phrase is confused with the meant meaning. Or, used for homonymous words.

Amphiboly – an argument's ambiguity is due to the grammatical construction of premises. **Accent** – fallacy that occurs when the accent of a premise is shifted and therefore delivers a meaning that was unintended. Normally found in stating quotations.

Composition – when the attributes of the parts of a whole are taken out of context from the rest of the argument.

Division – when arguing that what is true of a whole must also be true of its parts. Or when one argues from the attributes of a collection of parts to the attributes of the parts themselves.

Usage of definitions in arguments

Importance to understand meanings of words Eliminates the ambiguity of the argument Reduces the vagueness of the premises and conclusion Helps explain the argument theoretically Influences the attitudes of those people who need to be affected by the argument

5 rules of defining terms

State the essential attributes of the species. A definition should not define parts that are not relevant to the understanding of the term.

Should not be circular, or the predicate should not reiterate the subject.

Should not be too broad or too narrow

Must not be expressed in ambiguous, obscure, or figurative language. A definition should use clear and concise language that is understandable to everyone.

Should not be negative where it can be affirmative. Alternatively, a definition should be positively stated, because negativity can produce unwanted interpretatins.

5 types of definitions

Stipulative definitions are those where the assignment of meanings to new symbols is a matter of choice. For symbols mean nothing without a definition to back it up. **Lexical** definitions are those where the term being defined is not new but has an established usage. Lexical definitions can be either true or false, depending on the usage of the word.

Précising definitions is used to reduce the vagueness of a term. This means of defining a term helps not to redefine the word, but to put the term in words that are more understandable to the audience.

Theoretical definitions are those in which the most disputing over definitions occurs. Its urpose is to define a term with a scientific description. This often causes problems among many philosophers who are arguing the meaning of the terms, good, bad, evil etc.

Persuasive definitions are used to influence attitudes. This means of defining is often used emotively, and expressively to draw the attention of the audience.

Example 2: Your Choice of Scale Drawing by 2





To Double a Cube -- The Solution of Archytas

Here follows Heath's presentation:

"After Hippocrates had discovered that the duplication of the cube was equivalent to finding two mean proportionals in continued proportion between two given straight lines, the problem seems to have been attacked in the latter form exclusively. [, , ,]

Archytas

The solution of Archytus is the most remarkable of all, especially when his date is considered (first half of fourth century B. C.), because it is not a construction in a plane but a bold construction in three dimensions, determining a certain point as the intersection of three surfaces of revolution, (1) a right cone, (2) a cylinder, (3) a *tore* or anchor-ring with inner diameter *nil*. The intersection of the two later surfaces gives (says Archytas) a certain curve (which is in fact a curve of double curvature), and the point required is found as the point in which the cone meets this curve.

Suppose that AC, AB are the two straight lines between which two mean proportionals are to be found, and let AC be made the diameter of a circle and AB a chord in it.



Draw a semicircle with AC as diameter, but in a plane at right angles to the plane of the circle ABC, and imagine this semicircle to revolve about a straight line through A perpendicular to the plane of ABC (thus describing half a *tore* with inner diameter πil).

Next draw a right half-cylinder on the semicircle ABC as base; this will cut the surface of the half-tore in a certain curve.

http://www.uvw.com/pp/cubedbl.htm

Example 3 Computer programming for the best box.

The Box Problem



Example 4 Sticks, Spools, and a Vector Tree

The Vector Tree Project

I got the idea while looking at a mechnanical engineering book We Are looking for the center of gravity of a system of particles

R is the total weight. The position (X,Y,Z) is the balancing point and is found as follows:

Z= (wt on z axis) distance from Origin/R[.5*4/5] = .40" on z axisX= (wt on x axis) distance from Origin/R[2.40*6/5] = 2.40" on x axisY= (wt on y axis) distance from Origin/R[1.00*3/5] = .60" on y axis

Note: The 1.5 wt is at the origin and only affects R.

I used blocks of wood, 3/16", and 1/8" wood dowels, hooks, and string.

I do the sawing, students do the drilling and clipping.

I put the little sticks in spools that slide along the xz axis to the balancing point. Joining sticks gives the required point in space.

In practice I asked them to pick a point in space, then balance it out with weights.

Materials:



Place the weights







Example 5 Styrofoam and a Wizards Hat

This is called the wizards hat, Using a tack, string, scissors, Cut out the shape, then bring A and B together, and close the seam to get the wizzards hat.

String



Round 3: Group and Individual Cumulative Projects

Project 1: Smoothie on Over to Clay and Computer Project 2: Indoor/Outdoor Construction of the number 17

Project 1 Smoothie on Over to Clay and Computer

Smoothie on over

This is a project involving science, industrial art, and math. I got This idea from the book, Making Wooden Toys by Richard Blizzard.

I cut out a particular shape I liked. Something like a roller coaster or one can use the graph of the average temperature during the month of January in New Orleans. Another example: look up The American Teacher, vol 60, #9, Nov/Dec 1998, pg 692.

Ths project can be appreciated in the lower grades but its' ultimate use, I think is in calculus. The smoothness we identify with differentiability. The amount of wood or clay we identify with integration. The dips and peaks we identify with extreme. The place between the dips and peaks are called inflections The balancing point in the smoothie guy is called the centroid Parameterization of the x-axis wrt time.

An enrichment problem would be to name tis curve using LaGrange Polynomials where:

$$P(x) = \sum_{k=0}^{n} f(x_k) * L_{n,k}(x) \text{ where } L_{n,k}(x) = \prod_{\substack{i=0\\i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}$$

Larue I

James P. Larue G880TM Conference March 13, 1999

SMOOTHIE ON OVER

It's a project that can be done in grades 6-12, using algebra up to 'limiting' algebra (The Calculus). So let me demonstrate the "Smoothie Guy" in action as he covers his smooth and differentiable path.



25 cm wide, (0,25) is my domain 5 cm high, (0,5) is my range STIONS: What is the average height? How steep are the wave moguls? How does velocity or acceleration relate? What would you build something like this out of? (Wood, clay, styrofoam, plastic) Is this more Art, Physics, or Math? What is a centroid? (regarding the smoothie

a string example of giving time parameters to the x

which the they ado

Transfer shape of runners on to card or paper. Cut out patient and mansfer on to plywood.

GIUS



in front spand

naper , out

- 2 Cut out physical summers with coping saw. Both runners can be cut out in one operation if the two pieces of physicol are firmly clamped together before sewing.
- 3 Smooth all edges with glasspaper.
- Cut out spaces, which hold runners together. Cut statist front space.





- 5 Give and screw names to apacets checking alignment carefully.
- 6 Fix two nooks on front spacer. These catch clower when he reaches the top and return him back to the start.
- 7 Transfer shape of clown on to card or peper. Cut out pattern and transfer on to phywood. Cut out plywood.



ANTHIN ADDRES SAUMINES CLOWN MULICIPACION

DOWEL SPINOLE ROD

- 8 Only hate in clown and ht dowel spindle rold.
- 3 Crown must belance correctly. To adjust the head or feet as necessary.
- 10 Paint clown in bright colours
- 11 Rub chaik along runnin tracks. The clowin can now odifform.









DIM x(21), fx(21), y(21), p(21), c(21), e(21), e(21), ec(6, 6) m = 5 al n = m - 1LET yy = 01- tt = 1 PRINT "There will be"; m; "points or nodes to be entered one at a time." PRINT "Note that we will start with x(0) and end with x("; n; ") for" PRINT "notational reasons. fx(i) IS THE CORRESPONDING Y VALUE." PRINT " " PRINT " " PRINT "It may be necessary to parameterize inputs/outputs to work " PRINT "with my inputs of 0-5,5-10,10-15,15-20 corresponding to the" PRINT "four loops while all outputs are to range from -5 to 5." PRINT "Ofcourse one may go into the program and change vs and hs. " PRINT "" PRINT "" **PRINT** "In the first loop let x(0) = 0. In the second loop let x(0) = 5." **PRINT** "In the third loop let x(0) = 10 and in the fourth, 15. Your" PRINT " x(10)'s must end with 5, 10, 15, and 20 to maintain" PRINT "continuity of the general function which will be broken up into". PRINT "four sub-functions." PRINT " " 149 PRINT "after you have read and understood these instructions, press 3." INPUT t& IF t& = 3 THEN GOTO 151 GOTO 149 151 CLS FOR z = 1 TO 4 PRINT "This is loop number"; z; ". Your inputs must be from x = "; (z - 1) * 5 PR ? "if you wish to stop press 9" INPUT pp IF pp = 9 THEN GOTO 140 CLS FOR i = 0 TO n PRINT " " PRINT "enter x("; i; ")" INPUT x(i) y(i) = x(i)PRINT "enter fx("; i; ")" INPUT fx(i) NEXT i REM "product" FOR i = 0 TO n JAMES LARUE p(i) = 1FOR u = 0 TO n Louise S. McGeher IF u = i GOTO 22 p(i) = p(i) * (x(i) - x(u))2343 Prytania 2 NEXT U New Orleans, Louisians p(i) = fx(i) / p(i)EXT 1 10180 REM "find c(0)" 504-561-1224 (0) = 0 DR a = 0 TO n (0' = c(0) + p(a). sx. a REM "find c(1)" Ra=0 TO n a) = 0

Analysis

HARRE :

Test: Polynomial Interpolation

This part of the test is worth 40 points. Your individual test, taken in four days, is worth 40 points. Your one page summary of the SIRS article is worth 20 points and is to be handed in on the day of your individual test. It is to be typed and double-spaced.

This test must be turned in by each student. Place your answers on another sheet of paper. Show all work. Place graph on graph paper provided.

You may work in groups. This test will use two class periods. You may use notes, books, and calculators. You may <u>not</u> get help from anyone who is not in this class. Get to work.

. 10 points.

This test is based on table 1. Re-scale the domain to be 0 - 20. Adjust VS to be 100 (on line 50 of the smoothie program).

Find the polynomial for each of the usual four subintervals. Do not round the coefficients. Use the graph paper provided to copy over the graph from the screen.

2. 6 points.

For this question use your polynomials. Use the short-cut method to find p'(x) for interval #2. Find p'(8.2) and the equation of the tangent line. Graph it with the polynomial.

. 6 points.

a your graph to find the intervals where: p(x) is p(x) is p'(x) is p'(x) is

p'(x) is

4. 6 points

Use your graph to find all x such that p(x) = 0. In interval #1 only, find all x such that p'(x) = 0.

5. 6 points.

For this question use your polynomials. Use the partition [14,14.5,15,15.5,16] to find the area under the curve for the interval (14,16). If your curve drops below the zero line, then see me. You must produce both the upper and lower Riemann sums.

6. 6 points.

In interval #4 indicate by an arrow all the points of inflection on the graph.

Extra credit: 10 points

Translate your summary into the foreign language you have taken at this school. Get one of your teachers to correct it and sign it.

Project 2 Outdoor Construction of the number 17

Constructions:

A string, two tacks, straight edge, and pencil

Warm up:

A pencil, two tacks, and a string can be used to form an ellipse. Nice way to create the solar system.

