Blind Source Separation

for

Digital Modulation Schemes

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Abstract—An implementation of the Blind Source Separation problem as applied to digital modulation schemes (16-QAM, QPSK, 4-FSK) is described and simulated. The estimated unmixing matrix is obtained through Joint Approximate Diagonalization of Eigenmatrices (JADE). Unique challenges in the use of digital modulation schemes arise from phase rotations introduced by the mixing matrix and the whitening matrix applied during JADE pre-processing. Signal phase rotations are corrected using the M-power method, a non-data-aided algorithm for use with rotationally symmetric signal constellations. For the purposes of evaluating the proposed algorithm, all data generation and processing is performed using MATLAB code and Simulink modeling with symbol error rate (SER) utilized as a measure of system performance.

Index Terms—Blind Source Separation, Cumulant, Digital Modulation, Independent Component Analysis, Joint Diagonalization, Cumulant

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I. Introduction

The Blind Source Separation problem has been classically applied as a linear mixing of independent signals [1], [2], [3]. Analysis of speech signals (i.e. ,the "cocktail party" problem), EEG signals, or seismic vibrations are common practical applications of the blind separation problem [4], [5], [6].

BSS aims to differentiate signals that interfere with each other during propagation using only the mixtures that arrive at multiple receivers. Signal mixing is represented by an unknown mixing matrix A shown in Eqn. 1, where S is the original set of signals and X is the received set of mixtures. $X \in \mathbf{C}^{nxt}, A \in \mathbf{C}^{nxm}, S \in \mathbf{C}^{mxt}$ for *n* receivers, *m* transmitters, and *t* samples. No *a priori* information is assumed to be known about the sources, but they are taken to be statistically independent for the purpose of separation. The solution to the mixture problem lies in estimating the mixing matrix and solving the linear equation, Eqn. 2, where $\hat{A}^{+}$ indicates pseudo-inverse of the estimated mixing matrix A.

$$X(t) = A \cdot S(t) \tag{1}$$

$$\hat{S}(t) = \hat{A}^{+} X(t) \tag{2}$$

In contrast to conventional applications, this paper describes the use of the linear mixing model used in Blind Source Separation to recover and interpret digitally modulated communications signals. The independence criterion imposed on the source signals is still upheld under the assumption of statistically independent data-symbols sent from separate (independent) transmitters. The processing architecture where the BSS-ICA algorithm will be utilized is illustrated in Figure 1.

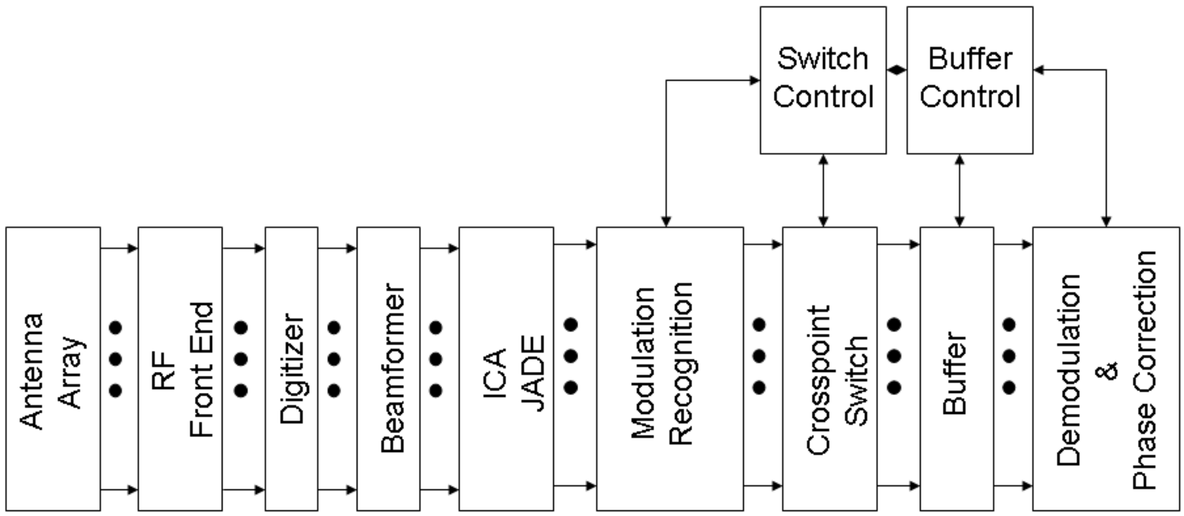


Figure 1: System Block Diagram

Referring to Figure 1, the basic inputs to the BSS-ICA algorithm arrive at the antenna array. The antenna array is interfaced to the RF Front End for preprocessing which would consist of Low Noise Preamplification (LNA), Automatic Gain Control (AGC), and possible frequency translation to an IF stage. The output of the IF stage is routed to the digitizer consisting of a bank of A/D converters. Beamforming, if utilized with the array, is performed in the digital domain for flexibility and the output of the beamformer is interfaced to the BSS-ICA algorithm. The BSS-ICA algorithm outputs the statistically independent un-mixed signal estimates to the Modulation Recognition System which ID's the specific modulation types which are then re-routed via the switch control to the crosspoint switch which connects the respective FIFO buffers to the correct channels of the Demodulation and Phase Correction processing unit.

II. Joint Approximate Diagonalization of Eigenmatrices

The solution to the BSS problem posed above lies in the estimation of an un-mixing matrix using the JADE algorithm. JADE is a higher order statistical method of independent component analysis (ICA). Cumulant tensors, which are composed by the cumulant generating function for X, defined in Eqn. 3 form the basis for separating mixed, statistically independent data.

$$g(t) = \log(E(e^{tX})) \tag{3}$$

Diagonal elements of a cumulant matrix characterize the distribution of a signal, while off-diagonal elements indicate statistical dependencies between signals [7]. JADE relies on the fact that the unmixed output signals are statistically independent only when cumulant tensors of all orders are diagonal matrices (i.e. they have minimal co-dependencies). Using Givens rotations to approximately diagonalize the cumulant matrices provides statistically independent output data up to a permutation and scaling change of the original signals. Diagonalizing the second order cumulant tensor is equivalent to decorrelation of the data (i.e. reducing the covariance matrix of the signals to the identity matrix). The decorrelation matrix is a transformation that whitens the mixed signals, and is denoted by *W*. The decorrelation matrix is composed of the eigenvectors of the covariance matrix of the mixed signals, and altered by a scaling factor. The properties of the decorrelation matrix W given a mixing matrix A are shown in Eqn. 4.

$$WR_y W^H = WAA^H W^H = I_n \tag{4}$$

The approximate diagonalization of third and fourth order cumulants requires an optimization criterion to measure the dependencies between signals. When the cumulant matrices are near diagonal, the signal co-dependencies are minimized, and this state represents the best value for the optimization criterion. The general Givens rotation matrix (Eqn. 5) for angle theta is used for diagonalization because of the orthogonality property, which causes no scaling change in the data. Further details on the JADE algorithm are given by Comon [8], Cardoso [9], et al.

$$G(\theta)=\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & & & \vdots & \iddots & \vdots \\ 0 & \cdots & \cos(\theta) & ...0... & \sin(\theta) & \cdots & 0 \\ \vdots & & ...0... & ...1... & ...0... & & \vdots \\ 0 & \ldots & -\sin(\theta) & ...0... & \cos(\theta) & \cdots & 0 \\ \vdots & \iddots & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \qquad (5)$$

III. JADE Algorithm with Complex Baseband Signals

A. Phase Rotation Problem

In baseband complex data, phase rotations of the signals of interest are introduced by the mixing matrix and during the whitening operation prior to JADE processing. These rotations are easily visualized as rotations of the de-mixed signal constellations in the complex plane (Figure 2). Given an arbitrary rotation of the signal constellation, the demodulation process can potentially produce 100% symbol error rates due to symbols being resolved incorrectly. To counter errors caused by phase rotation, the M-power method[10] is applied to Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) signals to correct the rotation. This corrected signal is still ambiguous due to the symmetry of PSK and QAM constellations, so each symmetric position is tested for accuracy to find the correct rotation and hence the minimal symbol error rate (SER). QAM and QPSK signals have symmetry over rotations of $\frac{\pi}{2}$, requiring the analysis of $\frac{2\pi}{\pi/2} = 4$ ambiguous cases. However, for M-PSK constellations, the signal constellation has symmetry over rotations of $\frac{2\pi}{M}$ for M symbols, which requires analysis of M ambiguous cases. This is an important consideration for the use of M-PSK with many symbols.

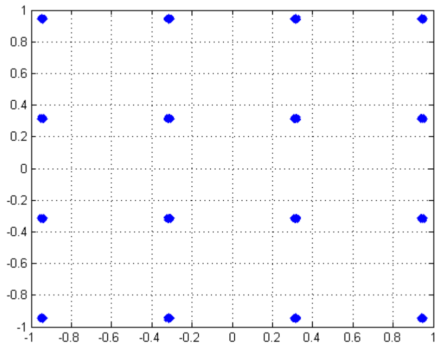


Figure 2a: QAM signal with no phase rotation

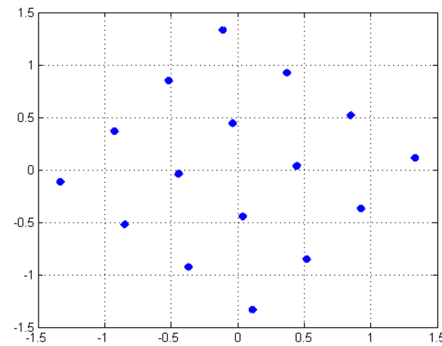


Figure 2b: QAM signal with 45 degree rotation

B. M-Power Method Solution

Although we have an estimate of the mixing matrix and the whitening matrix (the two complex-valued sources of the phase error in complex baseband signals), the rotation data cannot be extracted from this information due to the unknown permutation and sign change inherent to the data[11]. In light of this fact, a non-data-aided method of correcting phase rotation was adopted for its applicability to QAM and PSK signals and its availability as a Simulink library block. The M-Power method estimates phase error from complex valued symbols as shown in Eqn. 6.

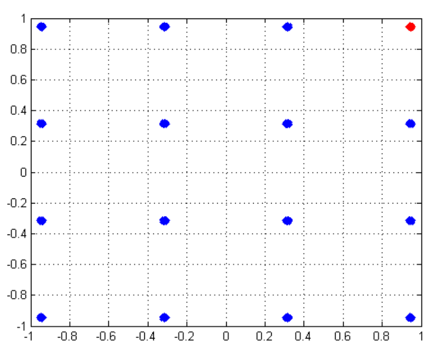
$$\theta_{Offset} = \frac{1}{M} angle\left\{\sum_{1}^{L} (x(k))^{M}\right\} \tag{6}$$

Here, x(k) is the complex representation of in-phase and quadrature components of the signal, L is the length of the signal, and the *angle* function converts a complex number to an angle. Since QAM and QPSK are both rotationally symmetric for $\theta = n(\pi/2)$, M is equal to 4 in both cases ($2\pi/(\pi/2) = 4$). Additionally, it can be shown that, using the M-Power method, the 16-QAM corrected signal always settles to a 45 degree offset (See Figure 2. approximately) instead of a 0 degree offset. This is easily corrected in the permutation testing by adjusting the phase correction by 45 degrees on QAM signals. This process corrects the signal to within a multiple of $\pi/2$, but that still allows for three rotations with near 100% error, and one correct rotation with minimal error.

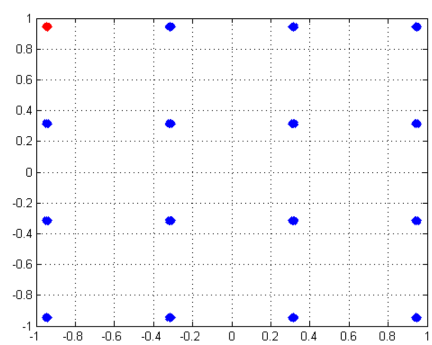
Our signals of interest (16-QAM and QPSK) only cause 4 possible permutations, so it is feasible to iteratively test the quality of the demodulated signals to infer which signal is correct. In the model, this is implemented as a minimization of symbol error rate (SER), which depends on the knowledge of the transmitted symbols. Of course, in reality that knowledge is not available, but the solution is to have a human analyst or automated analysis available to process the four permutations of demodulated output and decide which three signals have garbage information, and which signal is the correct rotation. For example, a human could listen to a digital audio signal and quickly determine which possibilities were garbage signals, and which one contains recognizable audio. Alternatively a machine might recognize that a signal does not match a predetermined data format, or that the data fails a checksum, etc. However, error detection and correction is outside the scope of this paper, and symbol error rate (SER) is chosen as a place-holder.

C. Signal Permutation Problem and Rotational symmetry solution

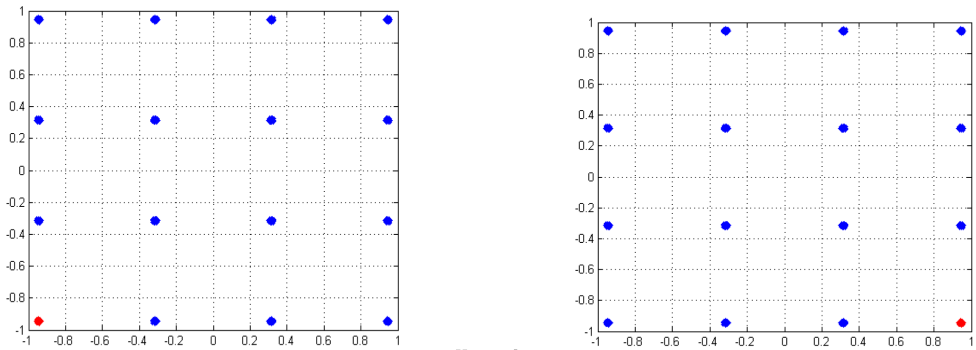
As implemented in the MATLAB and Simulink code at this time, a human analyst performs modulation recognition on the unmixed signals by viewing the constellation diagrams of the estimated signal set to properly order the signals for demodulation. An automated modulation recognition algorithm (e.g. the Agilent N682JE-MK1 system) could also perform the same task in this application (see Figure 1). The signal ordering is used to route the received signals to the appropriate demodulation scheme using the crosspoint switch and buffer illustrated in Figure 1. QAM and QPSK signals are rotationally adjusted using the M-Power method then demodulated in each rotationally symmetric position, in this case [0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$] (Figure 13~~Figure 3~~a-d). FSK signals are unaffected by phase rotation, so there is no need to adjust them prior to demodulation. Finally, each demodulation result is sent to a symbol error calculation using the transmitted symbols. The minimal symbol error rate is chosen to be the correct rotation, and a completely unmixed and demodulated signal is selected.



a)0



b)π/2



c)π d)-π/2

Figure 1~~3~~ (a-d): Four rotationally symmetric positions of a QAM signal labeled with applied rotation in radians. One symbol is highlighted for illustration purposes.

IV. JADE Algorithm with Real-Valued Carrier Frequency Signals

While many of the challenges lie with baseband complex signals, additional work was also carried out to show separation of mixed carrier signals. In the carrier signal model illustrated in Figure 4, a serial bit stream was converted to two parallel streams used to modulate the I and Q channels respectively.

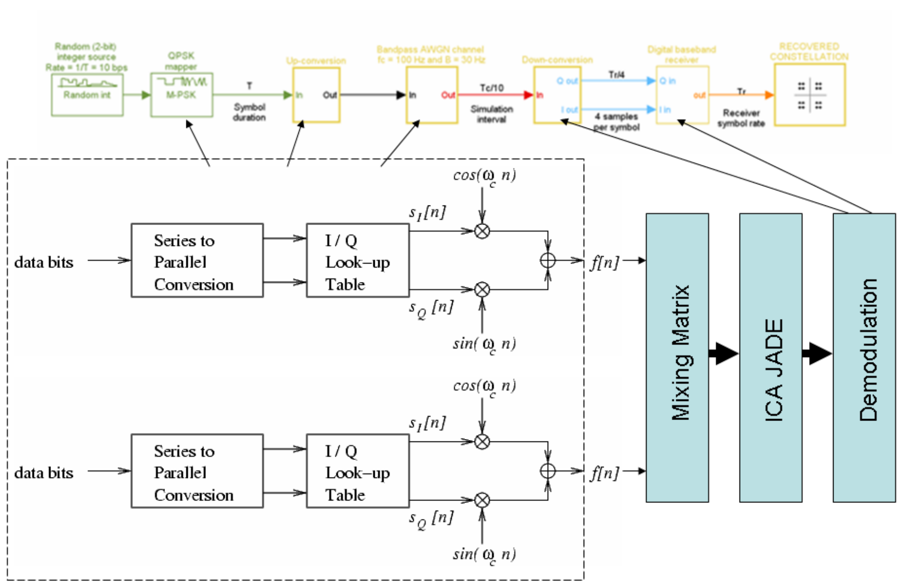
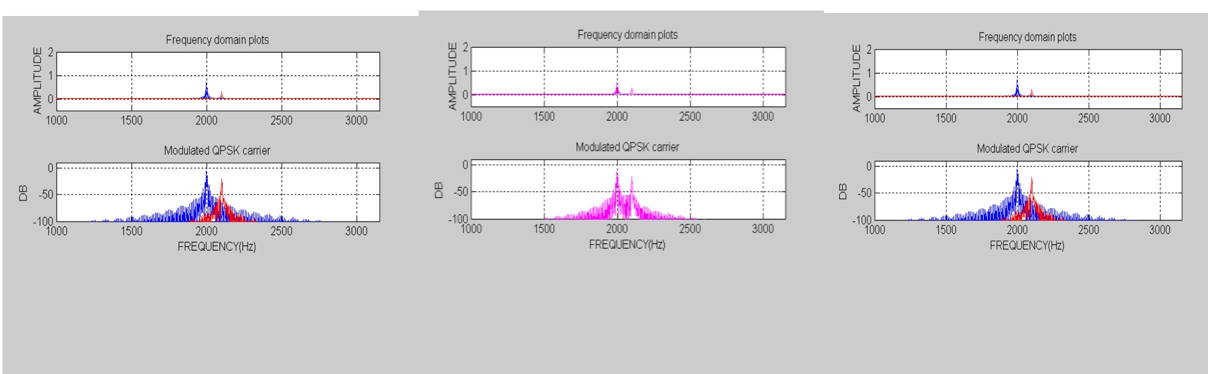


Figure 4: Upconversion Block Diagram

This simulation was computed for QPSK signals with closely spaced carrier frequencies (2 kHz and 2.1 kHz). Figure 5 shows a spectral plot before mixing, after mixing, and after JADE processing. The signals after JADE processing are again separated at the carrier frequency which can be determined by comparing the FFT plots before mixing and after JADE processing.

As an example of serial to parallel bitstream conversion, let the bitstream B={0,1,1,0,1,0,0,0,1,1}. The in phase bitstream B_I consists of the odd numbered bits 1,3,5,… and out of phase bitstream B_Q consists of the even numbered bits 2,4,6,… Therefore, for our example, B_I = {0,1,1,0,1} and B_Q = {1,0,0,0,1}. The I and Q bits are upconverted to the carrier frequency and summed to create a real-valued, modulated signal.



a. **b.** **c.**

Figure 5:(a) FFT of two modulated signals, one a 2kHz and one at 2.1kHz

(b) FFT of one mixture showing spikes at both 2 and 2.1 kHz

(c) FFT plot of two unmixed signals after JADE processing with clear separation

V. Algorithm Performance Analysis

A. Complex Baseband Signal Data

Complex data symbols can be unmixed using the JADE algorithm, but with the side-effect of permutation and scale changes, as well as the phase rotation problem. With these challenges met by phase rotation correction and modulation recognition (i.e., either human or automatic), symbol error rates drop to acceptable levels. **Error! Reference source not found.**~~Figure~~ 6 shows the importance of phase-correcting the digital baseband signals, as the demodulation often returns 100% incorrect symbols when a signal is significantly phase rotated. The figure is a plot of symbol error rate as a function of signal to noise ratio for QPSK signals at different phases of the unmixing process. The variability before phase correction, shown in the blue curve, is due to phase rotation. When a signal is phase rotated after JADE processing, the blue curve shows unacceptably high error rates, however the same plot can drop to 0% error if the signal is not out of phase after JADE processing. The variability in the blue curve is the motivation for phase correcting the unmixed signals, shown in green.

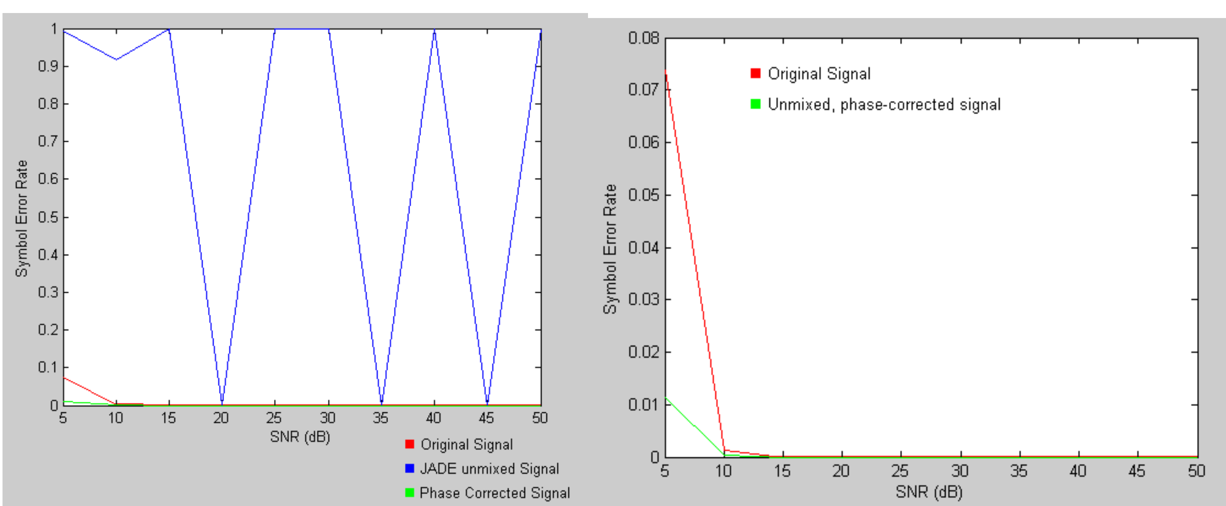


Figure 6: SER as a function of SNR for baseband QPSK signals

Figure 7 shows SNR versus SER for QAM signals. The JADE algorithm can recover signals at acceptable error rates even in low SNR scenarios, but phase correction is necessary to properly demodulate the unmixed signals. In Figures 6 and 7, comparison of the results before mixing (red) to the corrected JADE results (green) reveals error rates that are actually lower for the unmixed signals. This is likely caused by scaling and normalizing factors in the JADE algorithm which could eliminate some outliers and reduce the original error in a minimal way.

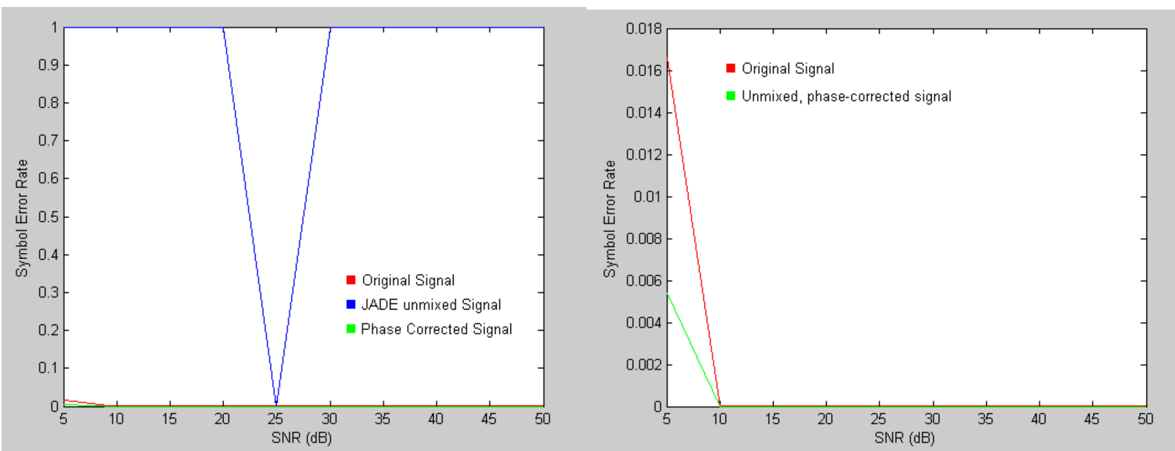


Figure 7: SER as a function of SNR for baseband QAM signals

B. Carrier Frequency Signal Data

Permutation and scaling are still challenges in real-valued carrier signal separation with JADE, but phase rotations are no longer a concern as they were for complex baseband signals. This is because we are working with real-valued data, where there are no complex values acting as phasors on the data during mixing and whitening (thus causing phase rotation). Any mixing and whitening transformations applied to carrier frequency data will be real-valued as well, causing only permutation and scaling changes. For carrier signals, permutation ambiguity can be resolved by locating the peak in the spectral plot, which indicates the carrier frequency of the signal. The signals can then be downconverted according to the frequency at which they are received. Modulation recognition is then performed as outlined for complex data.

Scaling ambiguity is the more troublesome problem in the form of sign changing, or inverting, the bitstream. If the received signals have been scaled by a negative number, arising from the normally distributed mixing matrix or the whitening matrix, the downconversion will produce an inverted bitstream (i.e. ones become zeros and vice versa). This problem is resolved by demodulating both the received bitstream and the inverse of the received bitstream (the only two possible signals for sign ambiguity) and choosing the signal with fewer errors.

Figure 8 shows JADE performance as a plot of symbol error rate versus signal to noise ratio for an FSK and QPSK signal, respectively. The curves in each plot show the symbol errors before mixing, after mixing, and after JADE processing at different signal to noise ratios. The blue curve represents mixed data and can be expected to have a consistently high error rate, regardless of SNR. Original and unmixed signals for FSK are unaffected by this level of noise and scaling, resulting in 0% error rates. The PSK shows high levels of error for low SNR, but the unmixed signal is nearly identical in error rate to the original signal, which is the best case we can expect.

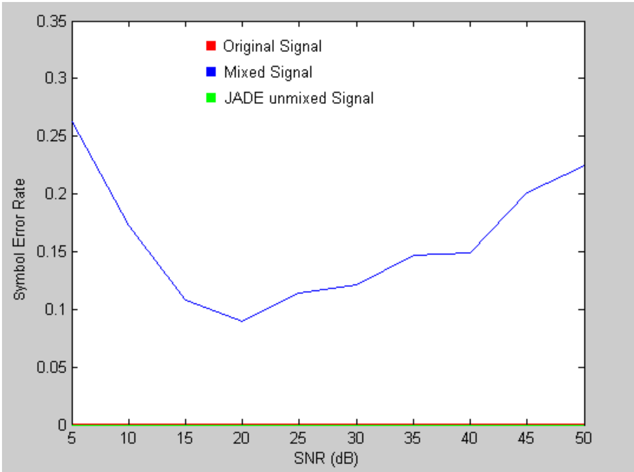


Figure 8a: FSK error rates vs. SNR

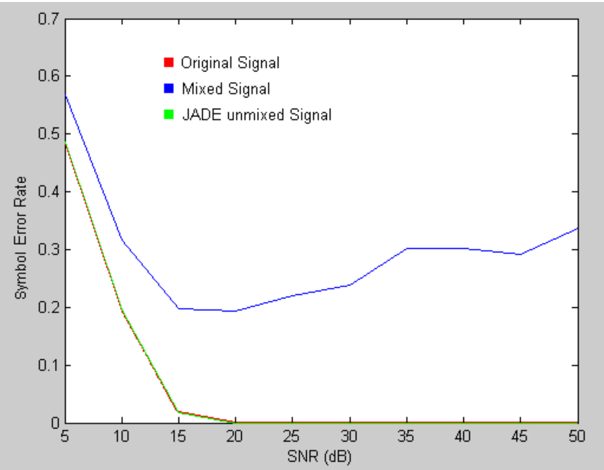
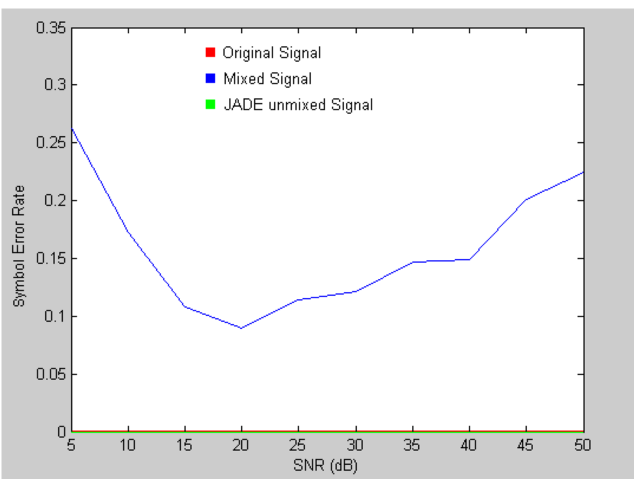
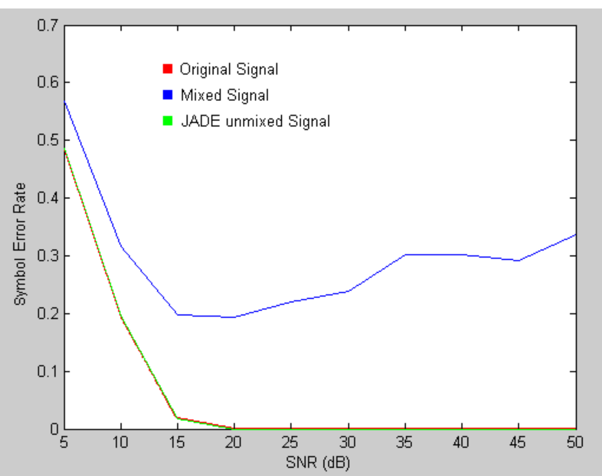


Figure 8b: QPSK error rates vs. SNR

The carrier signal

errors





at 0 dB are shown in Figure 9 as a graphical representation where each spike indicates a symbol error. An FSK signal is shown in the left side plots and a PSK signal on the right side plots. The top plots are the original errors after the application of additive white noise. The middle plots are taken after mixing, verifying substantially more errors, and the bottom plots in blue and red are the symbol errors after JADE processing. The teal lines in the bottom plots represent the difference between errors made at the pre-mix step and errors made after JADE unmixing. Again, the JADE unmixed signals have comparable symbol errors to the pre-mix signals, as evidenced in the teal plots. This indicates that even in noisy environments, JADE can be an effective unmixing tool.

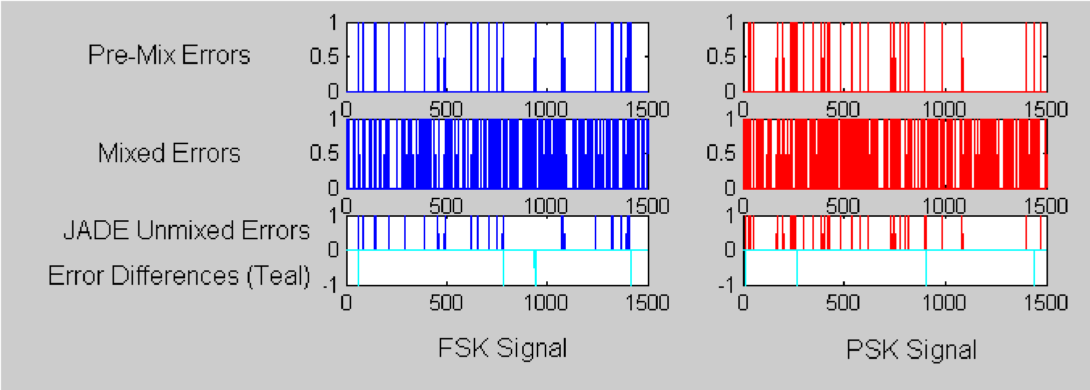


Figure 9: Bit errors represented as spikes (errors) deviating from 0 (no error) for FSK and QPSK at 0 dB SNR

Figure 10 shows the FFT plot of the PSK and FSK signal that generated the symbol error plots from Figure 9. Comparing the FSK (blue) and PSK (red), the peaks in the frequency spectrum show the closeness of the carrier frequency for the signals of interest. Despite the closely spaced carrier frequencies in a low SNR environment, JADE can still unmix with errors comparable to pre-mix demodulation (Figure 9).

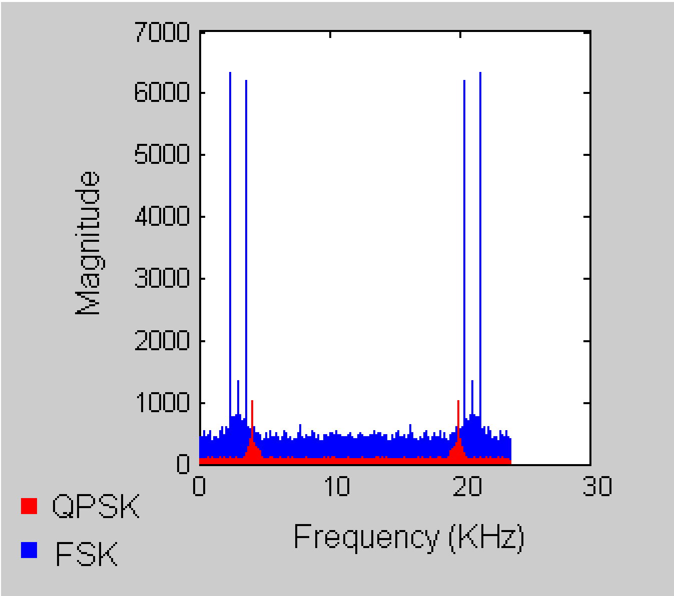


Figure 10: Two sided FFT plot of FSK and PSK signals showing closely spaced carrier frequencies, 3 kHz FSK and 4.2 kHz PSK

VI. Conclusions

An implementation of the Blind Source Separation problem as applied to digital modulation schemes (16-QAM, QPSK, 4-FSK) has been presented. The estimated unmixing matrix is obtained through Joint Approximate Diagonalization of Eigenmatrices (JADE). Signal phase rotations are corrected using the M-power method, a non-data-aided algorithm for use with rotationally symmetric signal constellations. All data generation and processing is performed using MATLAB code and Simulink modeling with symbol error rate (SER) as a performance measure.

Future research and applications of this work are to explore the performance of JADE in a multipath propagation environment and the effect of mixing convolved multipath signals with line of sight signals in the algorithm. Additionally, the Simulink model will be expanded to simulate more robust operation, including carrier signals, variable-dimension signal sets, and automated modulation recognition. The flowchart organization of the Simulink model allows specific elements such as the JADE algorithm to be easily lifted out and plugged into a separate model. This modular structure will be an effective tool for further algorithm development ans system integration in Simulink.

References

[1] A. Mansour, A. Kardec Barros, and N. Ohnishi, "Blind separation of sources: Methods, assumptions and applications," IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, vol. E83-A, no. 8, pp. 1498--1512, Special Section on Digital Signal Processing in IEICE EA, 2000.

[2] P. Bofill, "Underdetermined blind separation of delayed sound sources in the frequency domain," NeuroComputing, p. 627-641, Volume 55, Issues 3-4, October 2003.

[3] A. Mansour, M. Kawamoto, C. Puntonet, “A Time-Frequency Approach to Blind Separation of Under-Determined Mixture of Sources,” IASTED-ASM, Marbella, Spain, 3-5 September 2003.

[4] S. Choi and A. Cichocki, "Adaptive blind separation of speech signals: Cocktail party problem," in International Conference on Speech Processing, (Seoul, Korea), August 1997.

[5] Jung, T.-P., Makeig, S., McKeown, M. J., Bell, A. J., Lee, T.-W., Sejnowski, T. J. “Imaging brain dynamics using independent component analysis,” Proc. IEEE, 89(7): 1107-22, 2001.

[6] N. Thirion, J. Mars, and J. L. Boelle, "Separation of seismic signals: A new concept based on a blind algorithm," in Signal Processing VIII, Theories and Applications, Triest, Italy, pp. 85-88, Elsevier, September 1996.

[7] T. Blaschke, L. Wiskott, “An Improved Cumulant Based Method for Independent Component Analysis,” Proceedings of ICANN, pp. 1087-1093, 2002.

[8] L. Albera, P. Comon, “Joint Approximate Diagonalizations and Application to Virtual Arrays,” I3S Research Report, August 2003.

[9] J.F. Cardoso, A. Souloumiac, “Blind Beamforming for non Gaussian Signals,” IEE Proceedings-F, 140, p. 362-370, 1993.

[10] Moeneclaey, Marc, and Geert de Jonghe, "ML-Oriented NDA Carrier Synchronization for General Rotationally Symmetric Signal Constellations," IEEE Transactions on Communications, Vol. 42, No. 8, pp. 2531-2533, August 1994.

[11] Parra, L. and Spence, C. “Convolutive blind separation of non-stationary sources,” IEEE Transactions Speech and Audio Processing, pp. 320-7, 2000.